MATH 504 HOMEWORK 4

Due Monday, October 22.

Problem 1. *Suppose that κ is an inaccessible cardinal in L. Show that $L_\kappa = V_\kappa^L = V_\kappa \cap L$ and $L_\kappa \models ZFC + V = L$.*

Problem 2. *There exists $A \subset \omega_1$, such that $\omega_1 = \omega_1^{L[A]}$. (Hint: for each $\alpha < \omega_1$, find $A_\alpha \subset \omega$, such that $L[A_\alpha] \models \alpha$ is countable. Then look at $\alpha \mapsto A_\alpha$ and code it as a subset of ω_1.)*

Problem 3. *Show that $\diamondsuit$ implies the following:*

(1) *There is a sequence $\langle A_\alpha \mid \alpha < \omega_1 \rangle$, such that each $A_\alpha \subset \alpha \times \alpha$ and for all $A \subset \omega_1 \times \omega_1$, the set $\{\alpha < \omega_1 \mid A \cap (\alpha \times \alpha) = A_\alpha\}$ is stationary.*
(2) *There is a sequence of functions $\langle g_\alpha \mid \alpha < \omega_1 \rangle$, such that each $g_\alpha : \alpha \to \alpha$ and for all $g : \omega_1 \to \omega_1$, the set $\{\alpha < \omega_1 \mid g \restriction \alpha = g_\alpha\}$ is stationary.*

For a regular cardinal κ, define $\diamondsuit_\kappa$ to be the stement that there is a sequence $\langle A_\alpha \mid \alpha < \kappa \rangle$ with each $A_\alpha \subset \alpha$, such that for all $A \subset \kappa$, the set $\{\alpha < \kappa \mid A \cap \alpha = A_\alpha\}$ is stationary in κ. In particular, $\diamondsuit_{\omega_1}$ is just $\diamondsuit$.

Problem 4. *Let κ be a regular cardinal. Show that $\diamondsuit_\kappa$ implies that $2^{<\kappa} = \kappa$.*

Recall that Jensen's Covering lemma states that if $0^\sharp$ does not exists, then for every uncountable set $X \subset ON$, there is a set of ordinals $Y \in L$, such that $X \subset Y$ and $|X| = |Y|$.

Problem 5. *Suppose that $0^\sharp$ does not exists. Show that for every singular cardinal κ, $\kappa^+ = (\kappa^+)^L$ i.e. L computes successors of singulars correctly.*